Quality properties of linear and nonlinear abstract Schrödinger equations and applications

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In this talk, we consider the integral problem for linear abstract Schrödinger equation

$$i\partial_t u + \Delta u + Au = f\left(t, x\right), \; t \in \left[0, T\right], \; x \in \mathbb{R}^n, \tag{1}$$

$$u\left(0, x\right) = \varphi\left(x\right) + \int\limits_0^T \alpha\left(\sigma\right) u\left(\sigma, x\right) d\sigma,$$

and nonlinear abstract Schrödinger (NLAS) equation

$$i\partial_t u + \Delta u + Au + F\left(u\right) = 0, \; \left(t, \, x\right) \in \mathbb{R}_T^n, \tag{2}$$

$$u\left(0, x\right) = \varphi\left(x\right) + \int\limits_0^T \alpha\left(\sigma\right) u\left(\sigma, x\right) d\sigma,$$

where $\mathbb{R}_T^n = (0, T) \times \mathbb{R}^n$, α is a complex- valued function, A is a linear, F is a nonlinear operators in a Banach space E, Δ denotes the Laplace operator in $x \in \mathbb{R}^n$, $u = u(t, x)$ is a E-valued unknown function and $\varphi\left(x\right)$ a E-valued data function.

Here, regularity properties, Strichartz type estimates for solution of integral problem for linear and nonlinear abstract Schrödinger equations in vector-valued function spaces are obtained. The equation includes a linear operator A defined in a Banach space E, in which by choosing E and A we can obtain numerous classis of initial value problems for Schrődinger equations which occur in a wide variety of physical systems.

Remark 1.1. Note that particularly, by choosing $\alpha\left(\sigma\right)$ as a piecewise continuous function on $(0, T)$, the condition (1.2) can be expressed as the following multipoint nonlocal condition in time

$$u\left(0, x\right) = \varphi\left(x\right) + \sum_{k=1}^{m} \alpha_k u\left(\lambda_k, x\right),$$

where m is a positive integer, α_k are complex numbers and $\lambda_k \in \left(0, T\right)$.